A Note on Testing of Hypothesis

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Abstract : In this paper problem of testing of hypothesis is discussed when the samples have been drawn from normal distribution. The study of hypothesis testing is also extended to Baye's set up.

Keywords : Hypothesis, level of significance, Baye's rule.

1. Introduction

Let the random variable (r.v.) X have a normal distribution $N(\theta,\ \sigma^2)$, σ^2 is assumed to be known. The hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, $\theta_1 > \theta_0$ is to be tested. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta,\ \sigma^2)$ population. Let $\overline{X}\ (= \frac{1}{n}\sum_{i=1}^{n} X_i)$ be the sample mean.

By Neyman – Pearson lemma the most powerful test rejects H_0 at α% level of significance,

if $\frac{\sqrt{n}\left(\overline{X}-\theta_o\right)}{\sigma} \geq d_\alpha$, where d_α is such that

$$\int_{d_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} dZ = \alpha$$

If the sample is such that H_0 is rejected then will it imply that H_1 will be accepted?

In general this will not be true for all values of θ_1, but will be true for some specific value of θ_1 i.e., when θ_1 is at a specific distance from θ_0.

H_0 is rejected if $\frac{\sqrt{n}\left(\overline{X}-\theta_o\right)}{\sigma} \geq d_\alpha$

$$\text{i.e. } \overline{X} \geq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} \qquad (1)$$

Similarly the Most Powerful Test will accept H_1 against H_0

if $\frac{\sqrt{n}\left(\overline{X}-\theta_1\right)}{\sigma} \geq -d_\alpha$

i.e. $\quad \overline{X} \geq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}$ (2)

Rejecting H_0 will mean accepting H_1

if (1) $\Rightarrow$ (2)

i.e. $\quad \overline{X} \geq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} \quad \Rightarrow \quad \overline{X} \geq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}$

i.e. $\quad \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}} \leq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}}$ (3)

Similarly accepting H_1 will mean rejecting H_0

if (2) $\Rightarrow$ (1)

i.e. $\quad \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} \leq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}$ (4)

From (3) and (4) we have

$$\theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} = \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}$$

i.e. $\quad \theta_1 - \theta_0 = 2\, d_\alpha \frac{\sigma}{\sqrt{n}}$ (5)

Thus $d_\alpha \frac{\sigma}{\sqrt{n}} = \frac{\theta_1 - \theta_0}{2}$ and $\theta_1 = \theta_0 + 2 d_\alpha \frac{\sigma}{\sqrt{n}}$.

From (1) Reject H_0 if $\overline{X} > \theta_0 + \frac{\theta_1 - \theta_0}{2} = \frac{\theta_0 + \theta_1}{2}$

and from (2) Accept H_1 if $\overline{X} > \theta_1 - \frac{\theta_1 - \theta_0}{2} = \frac{\theta_0 + \theta_1}{2}$

Thus rejecting H_0 will mean accepting H_1

when $\quad \overline{X} > \frac{\theta_0 + \theta_1}{2}$.

From (5) this will be true only when $\theta_1 = \theta_0 + 2\, d_\alpha \frac{\sigma}{\sqrt{n}}$. For other values of

$\theta_1 \neq \theta_0 + 2\, d_\alpha \frac{\sigma}{\sqrt{n}}$ rejecting H_0 will not mean accepting H_1.

It is therefore, recommended that instead of testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1, \ \theta_1 > \theta_0$, it is more appropriate to test $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. In this situation rejecting H_0 will mean $\theta > \theta_0$ and is not equal to some given value θ_1.

But in Baye's setup rejecting H_0 means accepting H_1 whatever may be θ_0 and θ_1. In this set up the level of significance is not a preassigned constant, but depends on θ_0, θ_1, σ^2 and n.

Consider (0,1) loss function and equal prior probabilities ½ for θ_0 and θ_1. The Baye's test rejects H_0 (accept H_1)

$$\text{if} \quad \overline{X} > \frac{\theta_0 + \theta_1}{2}$$

and accepts H_0 (rejects H_1)

$$\text{if} \quad \overline{X} < \frac{\theta_0 + \theta_1}{2}.$$

[See Rohatagi p.463, Example 2]

The level of significance is given by

$$P_{H_0}\left[\overline{X} > \frac{\theta_0 + \theta_1}{2}\right] = P_{H_0}\left[\frac{(\overline{X} - \theta_0)\sqrt{n}}{\sigma} > \frac{(\theta_1 - \theta_0)\sqrt{n}}{2\sigma}\right]$$

$$= 1 - \Phi\left(\frac{\sqrt{n}(\theta_1 - \theta_0)}{2\sigma}\right)$$

where $\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} dZ$.

Thus the level of significance depends on θ_0, θ_1, σ^2 and n.

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